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A Study of Analyzing Greedy Approach for Fractional Knapsack Problem

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Abstract: The Knapsack Problem belongs to a large class of problems known as Combinatorial Optimization Problem. This problem is to maximize the obtained profit without exceeding the knapsack capacity. It is a very special case of the well-known Integer Linear Programming Problem. The purpose of this paper is to analyze several feasible solutions to a Fractional Knapsack Problem using greedy approach. Based on the knapsack algorithm to take different feasible solutions, in this set of feasible solutions, particular solution that satisfies the objective of the function. Such solution is called optimum solution. The optimum selection is without revising previously generated solutions. The greedy choices are made one after the other, reducing each given problem instance to smaller one. The greedy choices bring efficiency in solving the problem with the help of sub problems.

Keywords: Knapsack, Greedy Programming, Feasible Solutions.

I. INTRODUCTION

Greedy method is a straight forward method. This method is popular for obtaining the optimized solutions. In a greedy technique, the solution is constructed through a sequence of steps, each step examining a partially constructed solution obtained so far, until a complete solution to the problem is reached. At each step the choice made should be

Feasible – It should satisfy the problem's constraints.
Locally optimal – Among all feasible solutions the best choice is to be made.
Irrevocable – Once the particular choice is made then it should not get changed on subsequent steps.

In this method to select some solution from input, then to check whether the solution is feasible or not. The greedy method works on stages, at each stage only one input is generated at a time. Based on this input it decide whether it gives the optimum solution or not.

II. KNAPSACK PROBLEM

The Knapsack Problem is an example of a combinatorial optimization problem, which seeks for a best solution from among many other solutions. A thief robbing a store and can carry a maximum weight of W_i into their knapsack. There are n objects, from i=1,2,3…n. Each object has a particular weight W_i and obtains a particular profit P_i. What object should thief take? This version of problem is known as Fractional knapsack problem. The setup is same, but the thief can take fractions of items, meaning that the items can be broken into smaller pieces so that thief may decide to carry only a fraction of x_i of item i, where $0 \le x_i \le 1$. The aim is to fill the knapsack using various items so that the total weight of the items does not exceed the capacity of the knapsack. To choose only those objects that give maximum profit of the included objects. The total weight of selected objects should be $\le$ M.

Formula
Let M be the capacity of knapsack
Let X_i be the solution vector.

$$\sum_{1\le i\le n} P_i X_i$$
$$\text{and} \quad \sum_{1\le i\le n} W_i X_i \le M$$

Constraints $1 \le i \le n$ & $0 \le X_i \le n$,where the knapsack can carry the fraction X_i of an object i. Where P_i & W_i are the profit & weight are positive Numbers

Algorithm: Greedy Knapsack (m, n)

```
{
for i=1 to n
        {
                X[i] = 0.0;
                K= m;
                for i = 1 to n
                {
                        if(W[i] > K)
                                break;
                        X[i] = 1.0;
                        K = K- W[i];
                }
if(i≤ n )
                        X[i] = K/W[i];
        }
}
```

III. PROBLEM ANALYSATION

3.1 Knapsack Problem-1

Given n =4, M=8 ,(P_1,P_2,P_3, P_4) = (15, 10,9,5) , (W_1, W_2, W_3, W_4) = (1,5,3,4). The given problem can be solved by knapsack problem with Greedy method as shown below.

Feasible solution-1: Largest-profit strategy
To pick always the object with largest profit. If the weight of the object exceeds the remaining Knapsack capacity,

take a fraction of the object to fill up the Knapsack.
K=M=8 ,Put object 1 in the Knapsack.
P=15 Since $w_1 < M$ then $x_1=1$
K=8-1=7
Pick object 2 ,Since $w_2 < M$ then $x_2= 1$
K=7-5= 2
Pick object 3 ,Since $K< w_3$ then $x_3= K/w_3=2/3$.
Since the Knapsack is full then $x_4=0$.
The feasible solution is (1, 1,2/3,0).

Then calculate corresponding weight and profit

$\sum_{1\leq i\leq n} W_iX_i$ = (1*1 + 1*5+2/3*3+ 0*4)
= (1 +5+2 + 0) =8
$\sum_{1\leq i\leq n} P_iX_i$ = (1*15 + 1*10+2/3*9+ 0*5)
= (15 +10+6 + 0) =31.

Feasible solution-2: Smallest-weight strategy
To pick the object with the smallest weight. If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object to fill up the Knapsack.
K=M=8 , Pick object 1 in the Knapsack.
P= 15 ,Since $w_1 < M$ then $x_1=1$
K= 8-1 = 7
Pick object 3, Since $w_3 < M$ then $x_3=1$
K= 7-3 = 4
Pick object 4 ,Since $K<w_4$ then $x_4 =K/ w_4 =1$
Since the Knapsack is full then $x_2=0$.
The feasible solution is (1,0,1,1)

Calculate corresponding weight & profit

$\sum_{1\leq i\leq n} W_iX_i$ = (1*1 + 0*5+1*3+ 1*4)
= (1 +0+3 +4) =8
$\sum_{1\leq i\leq n} P_iX_i$ = (1*15 + 0*10+1*9+ 1*5)
= (15 +0+9 + 5) =29.

Feasible solution-3: Largest profit-weight ratio strategy
In order profit-weight ratios of all objects. $P_i/w_i >= (p_i+1)/(w_i+1)$ for $1 \leq i \leq n-1$. Pick the object with the largest p/w, if the weight of the object exceeds the remaining knapsack capacity; take a fraction of the object.
$P_1/w_1=15/1=15$ $P_2/w_2=10/5=2$ $P_3/w_3=9/3=3$
$P_4/w_4=5/4=1.25$
$P_1/w_1>=P_3/w_3>= P_2/w_2>= P_4/w_4$

K=M=8 , Pick object 1 in the Knapsack.

P=15 Since $w_1 < M$ then $x_1=1$
K= 8-1=7
Pick object 3 ,Since $w_3 < M$ then $x_3=1$
K= 7-3=4
Pick object 2
Since $K<w_2$ then $x_2=K/w_2=4/5$
Since the Knapsack is full then $x_4=0$
The feasible solution is (1,4/5,1,0)

Then calculate corresponding weight and Profit

$\sum_{1\leq i\leq n} W_iX_i$ = (1*1 + 4/5*5+1*3+ 0*4)
= (1 +4+3+ 0) =8
$\sum_{1\leq i\leq n} P_iX_i$ = (1*15 + 4/5*10+1*9+ 0*5)
= (15 +8+9 + 0) =32

Feasible solution-4: Smallest profit-weight ratio strategy
In order to profit-weight ratios of all objects. $P_i/W_i \leq (P_i+1)/(W_i+1)$ for $1 \leq i \leq n-1$. Pick the object with the smallest p/w,If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object.
$P_1/w_1=15/1=15$ $P_2/w_2=10/5=2$ $P_3/w_3=9/3=3$
$P_4/w_4=5/4=1.25$
$P_4/w_4 \leq P2/w_2 \leq P_3/w_3 \leq P_1/w_1$

K=M=8, Pick object 4 in the Knapsack.

P=5 Since $w_4 < M$ then $x_4=1$
K= 8-4=4
Pick object 2

Since $K<w_2$ then $x_2=K/w_2=4/5$
Since the Knapsack is full then $x_1=0$ and $x_3=0$
The feasible solution is (0,4/5,0,1)

Then calculate corresponding weight and Profit

$\sum_{1\leq i\leq n} W_iX_i$ = (0*1 + 4/5*5+0*3+ 1*4)
= (0+4+0+4) =8
$\sum_{1\leq i\leq n} P_iX_i$ = (0*15 + 4/5*10+0*9+ 1*5)
= (0 +8+0 +5) =13

The following table contains the Feasible solutions and with respective of their weight and profit values.

S.No	(X_1, X_2, X_3)	$\sum_{1\leq i\leq n} W_iX_i$	$\sum_{1\leq i\leq n} P_iX_i$
1	(1, 1,2/3,0)	8	31
2	(1,0,1,1)	8	29
3	(1,4/5,1,0)	8	32
4	(0,4/5,0,1)	8	13

In this set of feasible solutions, the largest profit-weight ratio strategy gives the maximum profit and hence it turns out to be optimum solution.

3.2 Knapsack Problem-2
Given n =5, M=6 ,(P_1,P_2,P_3, P_4, P_5) = (25,20,15,40,50), $(W_1, W_2, W_3, W_4, W_5)$= (3,2,1,4,5)

Feasible solution-1: Largest-profit strategy
To Pick always the object with largest profit. If the weight of the object exceeds the remaining Knapsack capacity, take a fraction of the object to fill up the Knapsack.
K=M=6 ,Put object 5 in the Knapsack.
P=50 Since $w_5 < M$ then $x_5=1$
K=6-5=1
Pick object 4, Since $K< w_4$ then $x_4= K/w_4=1/4$.
Since the Knapsack is full then $x_1=0$, $x_2=0$, $x_3=0$.
The feasible solution is (0,0,0,1/4,1).

Then calculate corresponding weight and profit

$\sum_{1\leq i\leq n} W_iX_i$ = (0*3 + 0*2+0*1+ 1/4*4+1*5)
= (0+0+0+1+5) =6
$\sum_{1\leq i\leq n} P_iX_i$ = (0*25+ 0*20+0*15+ 1/4*40+1*50)
= (0+0+0+10+50) =60

Feasible solution-2: Smallest-weight strategy
To pick the object with the smallest weight. If the weight of the object exceeds the remaining knapsack capacity,

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take a fraction of the object to fill up the Knapsack.
K=M=6
Pick object 3 in the Knapsack.
P= 15 ,Since $w_3 < M$ then $x_3=1$
K= 6-1 = 5
Pick object 2 , Since $w_2 < M$ then $x_2=1$
K=5-2 = 3
Pick object 1

Since $K<w_1$ then $x_1 = K/w_1 = 3/3 = 1$
Since the Knapsack is full then $x_4=0$, $x_5=0$.
The feasible solution is (1, 1, 1, 0, 0)

Calculate corresponding weight & profit

$\sum_{1\le i\le n} W_iX_i$ = (1*3 + 1*2+1*1+ 0*4+0*5)
= (3+2+1+0+0) =6
$\sum_{1\le i\le n} P_iX_i$ = (1*25+ 1*20+1*15+ 0*40+0*50)
= (25+20+15+0+0) =60

Feasible solution-3: Largest profit-weight ratio strategy
In order to profit-weight ratios of all objects. $P_i/w_i >= (p_i+1)/(w_i+1)$ for $1 \le i \le n-1$. Pick the object with the largest p/w, If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object.
$P_1/w_1=25/3=8.33$ $P_2/w_2=20/2=10$ $P_3/w_3=15/1=15$
$P_4/w_4=40/4=10$ $P_5/w_5=50/5=10$.

(i) Here some ratio values are same. So to take that same values in increase order of their weights. $P_3/w_3 >= P_2/w_2 >= P_4/w_4 >= P_5/w_5 >= P_1/w_1$

K=M=6
Pick object 3 in the Knapsack.

P=15 Since $w_3 < M$ then $x_3=1$
K= 6-1=5
Pick object 2 ,Since $w_2 < M$ then $x_2=1$
K= 5-2=3
Pick object 4, Since $K<w_4$ then $x_4 = K/w_4 = 3/4$
Since the Knapsack is full then $x_1=0$, $x_5=0$.
The feasible solution is (0,1,1,3/4,0)

Then calculate corresponding weight and Profit

$\sum_{1\le i\le n} W_iX_i$ = (0*3 + 1*2+1*1+ 3/4*4+0*5)
= (0+2+1+3+0) =6
$\sum_{1\le i\le n} P_iX_i$ = (0*25+ 1*20+1*15+ 3/4*40+0*50)
= (0+20+15+30+0) =65

(ii) Here some ratio values are same. So to take that same values in decreasing order of their weights .$P_3/w_3 >= P_5/w_5 >= P_4/w_4 >= P_2/w_2 >= P_1/w_1$

K=M=6
Pick object 3 in the Knapsack.

P=15 Since $w_3 < M$ then $x_3=1$
K= 6-1=5
Pick object 5

Since $K<w_5$ then x5 $=K/w_5=1$
Since the Knapsack is full then $x_1=0$, $x_2=0$, $x_4=0$.
The feasible solution is (0, 0, 1, 0,1)
Then calculate corresponding weight and Profit

$\sum_{1\le i\le n} W_iX_i$ = (0*3 + 0*2+1*1+ 0*4+1*5)
= (0+0+1+0+5) =6
$\sum_{1\le i\le n} P_iX_i$ = (0*25+ 0*20+1*15+ 0*40+1*50)
= (0+0+15+0+50) =65

Feasible solution-4: Smallest profit-weight ratio strategy
In order profit-weight ratios of all objects. $P_i/W_i \le (P_i+1)/(W_i+1)$ for $1 \le i \le n-1$. Pick the object with the smallest p/w, If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object.
$P_1/w_1=25/3=8.33$ $P_2/w_2=20/2=10$ $P_3/w_3=15/1=15$
$P_4/w_4=40/4=10$ $P_5/w_5=50/5=10$

(i). Here some ratio values are same. So to take that same values in increase order of their weights. $P_1/w_1 \le P_2/w_2 \le P4/w_4 \le P_5/w_5 \le P_3/w_3$

K=M=6, Pick object1 in the Knapsack.
P=25 Since $w_1 < M$ then $x_1=1$
K= 6-3=3
Pick object 2, Since $w_2 < M$ then $x_2=1$
K=3-2=1
Pick object 4, Since $K<w_4$ then $x_4=K/w_4=1/4$
Since the Knapsack is full then $x_3=0$ and $x_5=0$
The feasible solution is (1,1,0,1/4,0)

Then calculate corresponding weight and Profit

$\sum_{1\le i\le n} W_iX_i$ = (1*3 + 1*2+0*1+ 1/4*4+0*5)
= (3+2+0+1+0) =6
$\sum_{1\le i\le n} P_iX_i$ = (1*25+ 1*20+0*15+ 1/4*40+0*50)
= (25+20+0+10+0) =55

(ii). Here some ratio values are same. So to take that same values in decreasing order of their weights. $P_1/w_1 \le P_5/w_5 \le P4/w_4 \le P_2/w_2 \le P_3/w_3$

K=M=6
Pick object1 in the Knapsack.

P=25 Since $w_1 < M$ then $x_1=1$
K= 6-3=3
Pick object 5 in the Knapsack.

Since $K<w_5$ then $x_5=K/w_5=3/5$
Since the Knapsack is full then $x_2=0$, $x_3=0$, $x_4=0$.
The feasible solution is (1,0,0,0,3/5)

Then calculate corresponding weight and Profit

$\sum_{1\le i\le n} W_iX_i$ = (1*3 + 0*2+0*1+ 0*4+3/5*5)
= (3+0+0+0+3) =6
$\sum_{1\le i\le n} P_iX_i$ = (1*25+ 0*20+0*15+ 0*40+3/5*50)
= (25+0+0+0+30) =55

The following table contains the Feasible solutions and with respective of their weights and profit values.

S.No	(X_1, X_2, X_3)	$\sum_{1\le i\le n} W_iX_i$	$\sum_{1\le i\le n} P_iX_i$
1	(0,0,0,1/4,1)	6	60
2	(1,1,1,0,0)	6	60
3	(0,1,1,3/4,0)	6	65
	(0,0,1,0,1)	6	65
4	(1,1,0,1/4,0)	6	55
	(1,0,0,0,3/5)	6	55

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In this set of feasible solutions, the largest profit-weight ratio strategy gives the maximum profit. If the order of ratio values can be changed only the largest profit-weight ratio strategy gives the maximum profit and hence it turns out to be optimum solution.

IV. CONCLUSION

In this paper, we have taken different feasible solutions for knapsack problem using greedy approach. In this set of feasible solutions the largest profit-weight ratio strategy gives the maximum profit and hence it turns out to be optimum solution. The optimum selection is without revising previously generated solutions. So the knapsack problem using greedy technique is efficient for obtaining optimal solution.

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BIOGRAPHY

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